NEW THIRD-FAMILY FLAVOR PHYSICS
AND THE TOP MASS

B. HOLDOM[*]

Department of Physics, University of Toronto

Toronto, Ontario, CANADA M5S 1A7

ABSTRACT

A new massive gauge boson (X) coupling to the third family produces a tantalizing pattern of deviations away from the standard model. These include increasing Γ_b/Γ_h and decreasing the $\alpha_s(M_Z)$ extracted from Γ_h/Γ_ℓ. We indicate how the X boson may be related to a dynamical origin of the top mass.

When you ask those working on supersymmetric theories *why* they work on supersymmetric theories, they often refer to the predicted $\alpha_s(M_Z)$ using $\sin^2\theta_W$ as input. The minimal supersymmetric model gives $\alpha_s(M_Z)$ between 0.125 and 0.140 for superpartner masses between 0.1 and 1 TeV, when possible GUT threshold effects are ignored.[1] This is to be compared to the world average: $\alpha_s(M_Z) = 0.117 \pm .005$. The cleanest low-energy determinations of $\alpha_s(M_Z)$ are on the low side of the world average. Deep inelastic scattering and lattice calculations of quarkonia spectra yield $0.112 \pm .005$ and $0.115 \pm .002$ respectively. When compared to these numbers, the supersymmetry prediction is not completely compelling. The theoretical errors associated with α_s extracted from the hadronic τ width and from jet studies are less clear. (There is also a recent extraction of α_s from Υ production using sum rules, $0.109 \pm .001$,[2] but the true theoretical error is open to question here as well.)

There is also the LEP determination, and in particular the clean determination provided by a measurement of $R_\ell \equiv \Gamma_h/\Gamma_\ell$. This gives $\alpha_s(M_Z) = 0.125 \pm .005$. When averaged with the other determinations, it helps to move the world average higher. Instead of doing this, it is of interest to consider R_ℓ along with another quantity measured at LEP, $R_b \equiv \Gamma_b/\Gamma_h$. Here there is a well known 2% discrepancy with the standard model prediction, and I attribute this to the following positive shift in Γ_b.
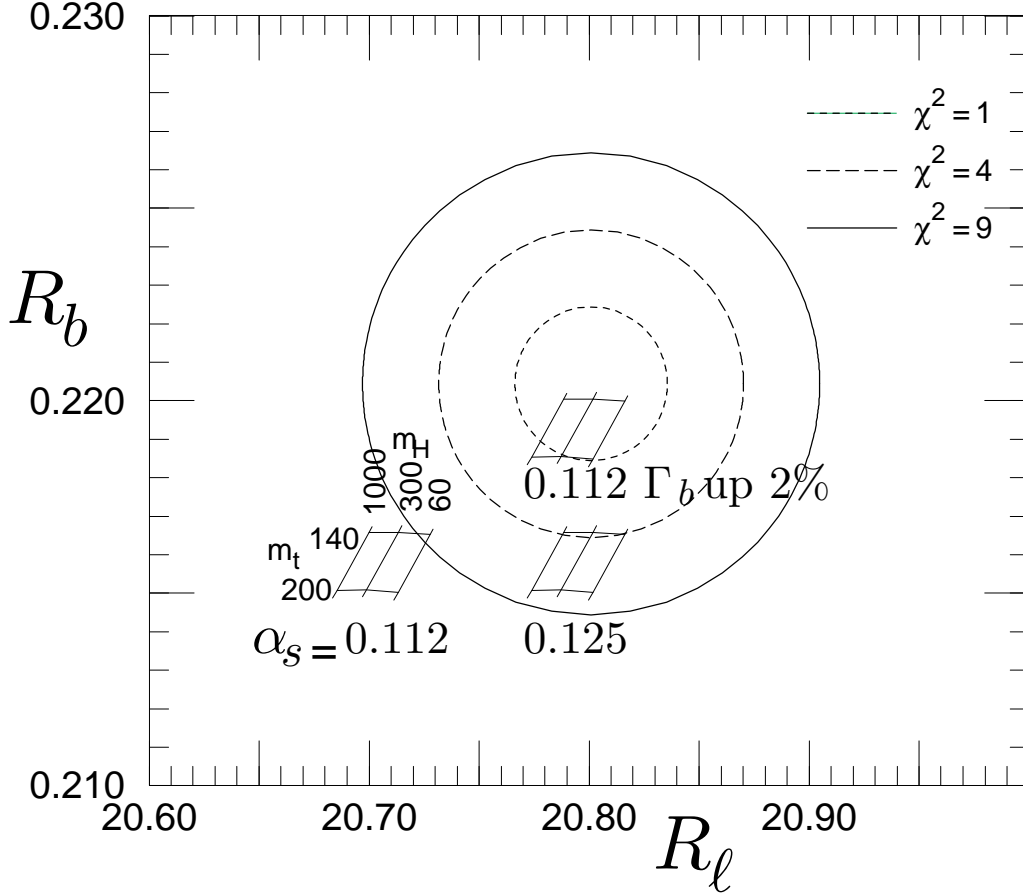
$$\delta\Gamma_b/\Gamma_b = 0.028 \pm 0.012 \qquad (1)$$

The point is that if this signals new physics in the $Zb\bar{b}$ vertex, then this same physics will increase Γ_h, and thus R_ℓ. To compensate, the α_s extracted from R_ℓ

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must decrease. A 2% increase in Γ_b corresponds to the following shift in α_s.[3]

$$\delta\alpha_s(M_Z) \approx -0.013 \qquad (2)$$

This would bring the α_s determined from R_ℓ in line with the other values mentioned above. Another way to see this is shown in the following figure in which R_b is plotted versus R_ℓ. (I use data reported in winter 1995 conferences.) The standard model prediction with $\alpha_s(M_Z) = 0.112$ is located between 3 and 4 σ away from the central value. Increasing $\alpha_s(M_Z)$ moves the standard model prediction horizontally. That may be compared with holding $\alpha_s(M_Z) = 0.112$ and instead increasing Γ_b by 2% above the standard model value. This brings about good agreement with the data, thus showing how the R_b and R_ℓ measurements both favor a new contribution to Γ_b.
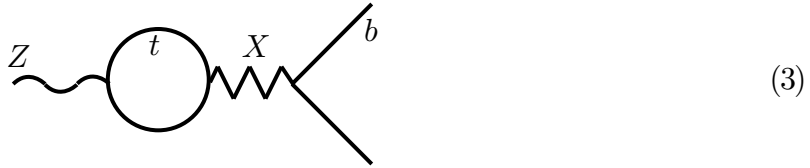

The question is whether this simple picture is supported by a global fit to all electroweak data. Until recently many global analyses in the literature did not allow $\alpha_s(M_Z)$ to vary along with the $Zb\bar{b}$ coupling g_Z^b. One of the first which did was provided by Matsumoto.[4] When S, T, g_Z^b and α_s were all allowed to vary he found that central value for α_s floated downward, $\alpha_s(M_Z) = 0.112 \pm .009$, in

agreement with the above discussion. For a more recent global fit with various assumptions about new physics see reference 5.

Although none of this constitutes a truly serious problem for the standard model, we are led to at least consider the possibility of new flavor physics associated with the third family. Would the latter be all that surprising? Perhaps not, at least in a dynamical symmetry breaking context. Consider the natural generation of quark and lepton masses due to strongly interacting gauge theories. The usual scenario has new fermions receiving mass of order 1 TeV, associated with the dynamical breakdown of electroweak symmetry. This mass is fed down to some quark q via new physics characterized by some scale Λ. The point is that the larger the m_q, the smaller the Λ; the mass scale of new interactions is inversely related to the mass of particles it couples to. Thus we should not be surprised if new flavor physics shows up first with the third family.

What does this new flavor physics look like? It is likely to involve a broken family gauge symmetry of some sort, and when gauge symmetries break one often has broken diagonal generators which correspond to massive $U(1)$ gauge fields. Thus a likely remnant of a broken family symmetry would be a massive gauge boson coupling to the third family but not lighter families. We will refer to this as the X boson.[3]

We can thus expect the following effect. Due to mixing between the Z and the X, for example through a top loop, the Z couplings to the b will be shifted.

$$(3)$$

We may deduce some required properties of the X by comparing this contribution to the standard model correction to the $Zb\bar{b}$ vertex, which also involves a t inside a loop. This latter contribution drives Γ_b down by about 2%, which is precisely the effect not seen in the data. This must then be more or less canceled by the new physics contribution. We see that a) the ratio g_X/M_X must be similar to that of the electroweak gauge bosons, b) the X should have an axial coupling to the t in order to produce mass mixing with the Z, and c) t and b can have the same sign axial coupling to the X. The latter is of interest if X is to originate from a family symmetry. Since we also expect that the X couples to the τ and ν_τ, the Z couplings to these leptons will also be shifted.

Is it likely that the X couples to the third-family fermions and to no other fermions? (We are ignoring small mass mixing effects.) It does not seem likely if one considers gauge anomalies and the fact that the X is emerging from a gauged family symmetry. This leads us to consider a fourth family and a X coupling to "third-family number minus fourth-family number". This generator easily emerges

from a family symmetry; it also clearly avoids gauge anomalies.

This fourth family can play a useful role; in fact we will assume that members of the fourth family develop the required ≈ 1 TeV dynamical masses. And we will require that this dynamics breaks not only the electroweak symmetry, but also the X boson gauge symmetry. For this to happen the fermion mass eigenstates are not the same as the states which have vector couplings to the X. In particular, let q and $\underline{q}$ be the two quark doublets with equal and opposite vector X charges. Suppose the condensate which forms is $\langle \bar{\underline{q}}_L q_R \, + \, h.c. \rangle \, \neq \, 0$. This defines the mass eigenstates for the fourth family quarks with $m_{t'} \approx m_{b'} \approx 1$ TeV, whose Dirac fields (t', b') are each composed of $[\underline{q}_L, q_R]$.

The result is that the X has *axial* couplings to (t', b'). This in turn implies that vacuum polarization graphs involving the t' and b' will produce mass for both the Z and the X. And this determines the coupling to mass ratio for the X.[3]

$$\frac{g_X}{M_X} \; = \; \frac{e}{4 c s M_Z} \tag{4}$$

The third-family quarks (t, b) are then composed of $[q_L, \underline{q}_R]$, which implies that (t, b) also have axial X couplings. For the τ and τ' on the other hand, we take them to be composed of $[\tau_L, \tau_R]$ and $[\underline{\tau}_L, \underline{\tau}_R]$ respectively, which implies that the τ has vector couplings to the X. The reason for this choice is made clear below. The result is that the X couples to the following third-family current.

$$J_\mu^X \; = \; \bar{t} \gamma_\mu \gamma_5 t \; + \; \bar{b} \gamma_\mu \gamma_5 b \; + \; \bar{\tau} \gamma_\mu \tau \; + \; \bar{\nu}_{\tau L} \gamma_\mu \nu_{\tau L} \tag{5}$$

By comparing the Z-X mixing diagram involving the t loop to the Z mass diagram involving the t' and b' we find that the Z couplings to the third family are shifted by an amount $\delta g_Z Z^\mu J_\mu^X$ where[3]

$$\delta g_Z \; \approx \; -\frac{e}{8 \, c \, s} \left(\frac{m_t}{m_{q'}} \right)^2 \tag{6}$$

We may express the various shifts in the Z couplings in terms of the asymmetry parameters $\mathcal{A}_f = 2 g_v^f g_a^f / (g_v^{f2} + g_a^{f2})$ and the partial decay widths $\Gamma_f \propto g_v^{f2} + g_a^{f2}$. For the charged leptons the axial coupling is much larger than the vector coupling. Thus the observed similarity between Γ_e, Γ_μ, and Γ_τ places a strong constraint on g_a^τ. This is the reason we have chosen the τ to have vector couplings to the X boson.

Besides the shifts in the quantities Γ_b and α_s as we have described, we also have the shifts in the following table along with the relevant observable. The latter are chosen[3] specifically to look for universality breaking corrections involving the third family, and are quite insensitive to possible oblique corrections. In the case of $\delta \mathcal{A}_\tau$ there are two such independent observables. Note also that certain systematic errors cancel in the ratios.

$\dfrac{\delta \mathcal{A}_\tau}{\mathcal{A}_\tau}$	$\left(\dfrac{A_{FB}^{0\tau}}{A_{FB}^{0e,\mu}} \quad \text{and} \quad \dfrac{3\,P_\tau}{4\,P_\tau^{FB}} \right) \;\Rightarrow\; \dfrac{\mathcal{A}_\tau}{\mathcal{A}_e}$
$\dfrac{\delta \Gamma_{\nu_\tau}}{\Gamma_{\nu_\tau}}$	$\dfrac{\Gamma_{\text{inv}}}{\Gamma_\ell} \;=\; \dfrac{\Gamma_{\nu_e} + \Gamma_{\nu_\mu} + \Gamma_{\nu_\tau}}{\Gamma_\ell}$
$\dfrac{\delta \Gamma_\tau}{\Gamma_\tau}$	$\dfrac{\Gamma_\tau}{\Gamma_{e,\mu}}$

The following table shows the results. Our estimate of Z-X mixing produces shifts in Γ_b and α_s of the desired magnitude, as discussed above. For $\mathcal{A}_\tau$ the X boson produces a large shift; this is not inconsistent with the average of the two experimental determinations, which in turn are not in good agreement with each other. For $\Gamma_{\nu\tau}$ and Γ_τ the experimental shifts are consistent with zero, but they are also not inconsistent with the X boson.

	Measurement	X boson
$\delta\Gamma_b/\Gamma_b$	$+0.028 \pm 0.012$	$+0.021$
$\delta\alpha_s(M_Z)$		-0.014
$\delta\mathcal{A}_\tau/\mathcal{A}_\tau$	$+0.32 \pm 0.19$ $+0.02 \pm 0.09$	$+0.21$
$\delta\Gamma_{\nu\tau}/\Gamma_{\nu\tau}$	-0.014 ± 0.023	-0.015
$\delta\Gamma_\tau/\Gamma_\tau$	$+0.003 \pm 0.004$	$0.0022 + 0.0015$

We also have to consider other corrections to the Z vertex where an X is exchanged between the two third-family fermions.[6] We may write the shifts in the Z couplings in the following way.

$$\delta g_Z \;=\; \text{constant} \;+\; \mathcal{O}\!\left(\frac{q^2}{M_X^2}\right) \tag{7}$$

The Z-X mixing contributes to the constant term, while the additional vertex corrections contribute to the q^2 term, where q is the 4-momentum entering the vertex. These latter corrections are then suppressed, and they are only important in the case of Γ_τ where they produce the term 0.0015 in the $\delta\Gamma_\tau/\Gamma_\tau$ entry of the table.

We motivated the existence of new flavor physics from the large size of the top mass. In fact the existence of an X boson with the properties we have described originated in a model[7] in which the top mass arose in a dynamical context different

from extended technicolor. Such a model must not only explain the top mass, but also explain why the electroweak breaking condensates, in this case the four family quarks, preserve isospin symmetry to good approximation. The trick is to keep the $SU(2) \times U(1)$ breaking physics of the condensate distinct from the isospin breaking physics responsible for the t mass. This is where ETC theories run into problems.[8] If some ETC interaction is able to produce an operator of the form $\overline{U}U\bar{t}t$ with a coefficient much larger than the one for $\overline{D}D\bar{b}b$, to produce a large top mass, then the same interactions are very likely to produce a $\overline{U}U\overline{U}U$ operator with a coefficient much larger than the one for $\overline{D}D\overline{D}D$. These latter operators affect the size of condensates through the gap equations, and in particular they make it very difficult to understand why $\langle \overline{U}U \rangle \approx \langle \overline{D}D \rangle$.

Our model introduces some new ingredients in an attempt to overcome this problem. One is to replace technicolor with "hypercolor", with the main distinction being that hypercolor breaks at a TeV via the same condensate which breaks $SU(2) \times U(1)$. The X boson is a broken diagonal generator of hypercolor. The third and fourth families, originally part of hypercolor multiplets, now emerge as singlets under the unbroken subgroup of hypercolor. The condensate involves the fourth family as we have already described. If hypercolor is a walking theory then some four-hyperfermion operators originating at higher scales can be expected to be strongly enhanced. Among such operators are those which break isospin (but not of course $SU(2) \times U(1)$). And among these operators is one, which we do not give here, which contains $\bar{t}q'\bar{q}'t$ but not $\bar{b}q'\bar{q}'b$ or $\bar{q}'q'\bar{q}'q'$. Because it is strongly enhanced by hypercolor, when combined with the fourth family condensate it can generate a large top mass. It does not produce a b mass nor does it contribute to the $Zb\bar{b}$ vertex. And as well, the presence of this operator by itself is consistent with $\langle \bar{t}'t' \rangle \approx \langle \bar{b}'b' \rangle$. Other more dangerous operators may be present, but because of their different structure they are not enhanced by hypercolor scaling effects nearly so strongly. One of the main features of this picture is that isospin breaking originates dynamically, via $SU(2)_R$ breaking, at a scale of order 100–1000 TeV. For more details see references 3, 6, and 7.

Acknowledgements

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